

## **Title: “As the World Turns”**

### **Brief Overview:**

The students will be able to reflect, rotate and translate a point, triangle, and a rectangle using the Geometer's Sketchpad. The students will be able to define each motion. The students will be able apply geometric motions to solve a real-life situation.

### **NCTM 2001 Principles for School Mathematics:**

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

### **Links to NCTM 2001 Standards:**

- **Content Standards**

#### **Number and Operations**

The students will be able to understand numbers, ways of representing numbers, relationships among numbers, and number systems.

#### **Algebra**

The students will be able to represent and analyze mathematical situations and structures using algebraic symbols.

#### **Geometry**

The students will be able to analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships. They will be able to specify locations and describe spatial relationships using coordinate geometry and other representational systems; apply transformations and use symmetry to analyze mathematical situations; and be able to use visualization, spatial reasoning, and geometric modeling to solve problems.

### **Measurement**

The students will be able to apply appropriate techniques, tools, and formulas to determine measurements.

#### **• Process Standards**

### **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**

These five process standards are integrated throughout this learning unit and are required in daily analytical problem solving.

The students will use the Geometer's Sketchpad to demonstrate geometric motions and to introduce technology into the learning unit.

### **Links to Maryland High School Mathematics Core Learning Units:**

#### **Geometry, Measurement, and Reasoning**

##### **• 2.1.1**

The students will describe the characteristics of geometric figures and will construct or draw geometric figures using technology and tools.

##### **• 2.1.3**

The student will use transformations to move figures, create designs and demonstrate geometric properties.

##### **• 2.1.4**

The students will validate properties of geometric figures using appropriate tools and technology.

##### **• 2.2.2**

The students will solve problems using two-dimensional figures and/or right triangle trigonometry.

### **Grade/Level:**

Grades 9–12, Geometry

### **Duration/Length:**

Two or three class periods, approximately 45-55 minutes in length

### **Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- Using Geometer's Sketchpad to draw basic geometric figures

- Knowledge of definitions of basic geometric figures
- Knowledge of the basic “undefined terms of geometry”
- Knowledge of the coordinate plane

### **Student Outcomes:**

Students will:

- be able to define rotation, translation and reflections as they apply to geometric motion.
- use the Geometer’s Sketchpad to demonstrate rotations, translations and reflections.
- application of these concepts to solve real-life problems.

### **Materials/Resources/Printed Materials:**

- Textbook
- Geometer’s Sketchpad
- Geometer’s Sketchpad Instruction Sheets
- Vocabulary Sheet
- Student Activity Sheets
- Student Homework Assignment
- Student Assessment Sheet

### **Development/Procedures:**

- 1) Introduction of new concept of geometric motion.
- 2) Definition of motion in every day life.
- 3) Definition of specific geometric motions with a handout.
- 4) Discussion of specific motions with examples.
- 5) Student activity to review concept of geometric motion.
- 6) Geometer's Sketchpad activity with instructions.
- 7) Homework assignment to review and assess student achievement.
- 8) Review of concepts through questioning and assessing homework.
- 9) Student formal assessment order.

**Assessment:**

This is an activity-based unit with several worksheets that students will complete and have checked. Oral questioning techniques will also be a valuable assessment tool in this unit. The students also will be using Geometer's Sketchpad which will enable visual and hands-on assessment. A formal assessment will be given at the end of the learning unit with a scoring guide provided.

**Extension/Follow Up:**

Continuing the concept of geometric motion, students will learn dilations, symmetry, and tessellations. These concepts will complete geometric motions. Tessellations will tie all the motions together using composition of transformations. Geometer's Sketchpad has several excellent activities to demonstrate the combination of these concepts.

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## **Assessment: Teacher's Guide**

### **Introduction**

Students will discover properties of geometric motions using vocabulary, activity sheets, and Geometer's Sketchpad. They will develop mastery learning through completion of worksheets, computer activities and homework assignments. A formal assessment will be given at the end of the learning unit.

### **Objectives Covered**

- Gain a visual and geometric understanding of specific geometric motions.

### **Tools/Materials Needed for Assessment**

- Geometer's Sketchpad
- Activity Worksheets
- Formal Assessment

### **Administering the Assessment**

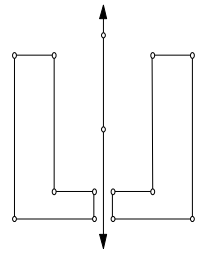
Students will be introduced to geometric motions by relating it to motion in the real world. These motions will then be defined geometrically and demonstrated visually. The motions will be preliminarily assessed by giving student a worksheet to test for general understanding. The teacher can assess understanding by checking the worksheet and proceeding to computer activity through Geometer's Sketchpad. This activity will be self-paced and allow for visual assessment and one-on-one assistance. Once students have completed the Sketchpad activity, they will be given a general homework assignment to test overall understanding. The homework will be reviewed which will allow time for student questioning. A formal assessment will then be administered which will require one class period.

### ***Rubric for Brief Constructed Responses***

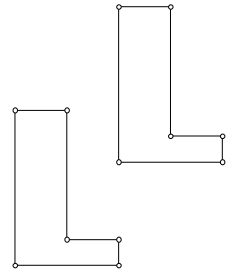
- 3- Graphs are complete and accurate (labels etc.).
- 2- Graphs are complete and mostly accurate.
- 1- Graphs are mostly complete and mostly accurate.
- 0- Graphs are incomplete and inaccurate.

## DEFINITIONS OF GEOMETRIC MOTIONS

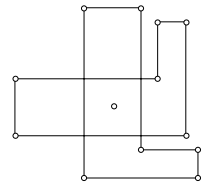
Reflections: also known as a flip, it is a mirror image of itself. The reflected object (Image) will have the opposite orientation of the original object (Preimage).



Translations: a sliding motion or change in position. A translation transforms the original object (Preimage) the same distance and the same direction. The motion is described by a vector  $\langle x, y \rangle$  where  $x$  represents a horizontal change and  $y$  represents a vertical change.



Rotations: the turning of an object, rotating the original object (Preimage) does not change the orientation of the image. To rotate an object, one needs to designate a center of rotation and an angle and direction of rotation.



Tessellations: patterns formed by repeating shapes to fill a plane without gaps or overlaps. Combining translations, reflections, and rotations creates these patterns.

Transformation: a geometric movement, change in position, shape, or size of a figure. It moves each point of the original object (preimage) to a new position, shape or size called the new object (image).

Mapping: one-to-one correspondence from the preimage to the image.

Dilation: a transformation, in which the size of an object is changed, either larger or smaller, by a scale factor ( $k$ ) where  $k > 1$  it is an enlargement and  $k < 1$  it is a reduction.

Preimage: original object.

Image: transformed object.

## STUDENT ACTIVITY WORKSHEET

1. Define geometric motions:\_\_\_\_\_

\_\_\_\_\_

2. Define translation:\_\_\_\_\_

\_\_\_\_\_

3. Define rotation:\_\_\_\_\_

\_\_\_\_\_

4. Define reflection:\_\_\_\_\_

\_\_\_\_\_

5. Translations are sometimes called a/an \_\_\_\_\_.

6. Rotations are sometimes called a/an \_\_\_\_\_.

7. Reflections are sometimes called a/an \_\_\_\_\_.

8. A preimage is :\_\_\_\_\_

\_\_\_\_\_

9. An image is:\_\_\_\_\_

\_\_\_\_\_

10. Artists often use translations, reflections, and rotations in designs called\_\_\_\_\_.

11. Given  $\langle -5, 3 \rangle$ , explain which motion and what it does to the motion:\_\_\_\_\_

\_\_\_\_\_

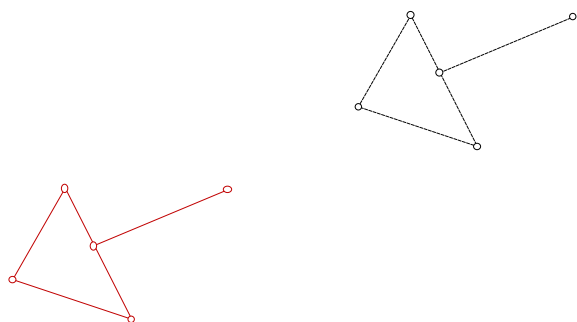
\_\_\_\_\_

12. The moving of each point of an original figure called the preimage to a new figure called the image is called\_\_\_\_\_.

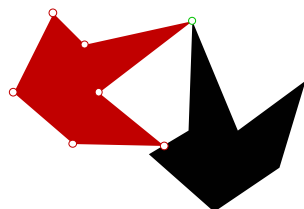
13. A mapping is a\_\_\_\_\_.

14-17. Identify each motion as a translation, reflection, or rotation.

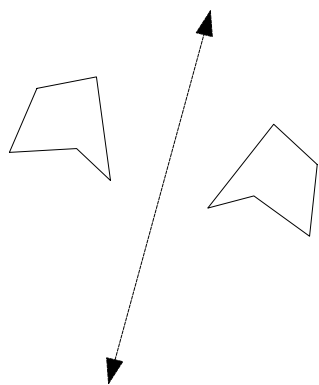
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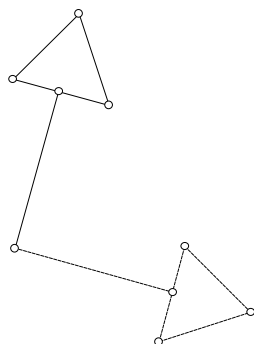
15.



16.



17.



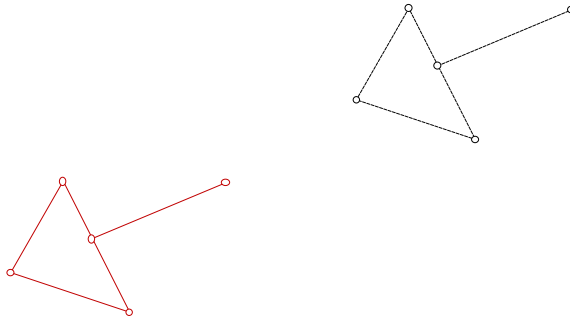


**ANSWER KEY**  
**STUDENT ACTIVITY WORKSHEET**

1. Define geometric motions: Change in position, shape, size, or direction of an object.  
A reflection, translation, rotation, or dilation of an object.
2. Define translation: A sliding motion or change in position.  
\_\_\_\_\_
3. Define rotation: The turning of an object. Rotating the preimage object does not  
change the orientation of the image.
4. Define reflection: Also known as a flip, it is a mirror image of itself.  
\_\_\_\_\_
5. Translations are sometimes called a/an "sliding" motion.
6. Rotations are sometimes called a/an "turning" of an object.
7. Reflections are sometimes called a/an "flip".
8. A preimage is: original object  
\_\_\_\_\_
9. An image is: transformed object  
\_\_\_\_\_
10. Artists often use translations, reflections, and rotations in designs called tessellations.
11. Given  $\langle -5, 3 \rangle$ , explain which motion and what it does to the motion: -5 represents  
a horizontal change (5 units to the left) and 3 represents a vertical change (3 units  
upward)
12. The moving of each point of an original figure called the preimage to a new figure  
called the image is called transformation.
13. A mapping is a one-to-one correspondence from the preimage to the image.

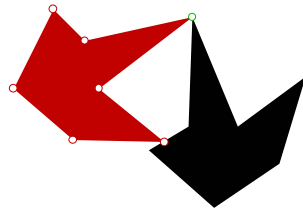
14-17. Identify each motion as a translation, reflection, or rotation.

14.



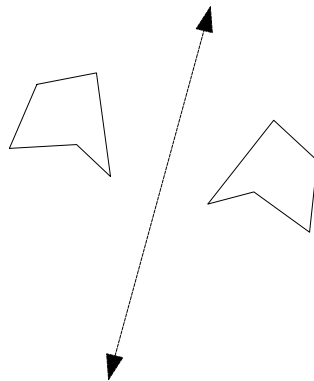
Translation

15.



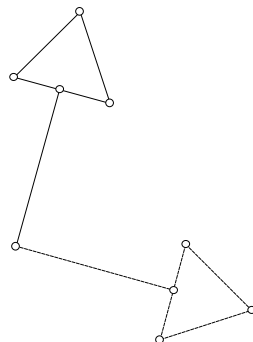
Rotation

16.



Reflection

17.



Rotation

## **GEOMETER'S SKETCHPAD STEP-BY-STEP INSTRUCTIONS**

Start up the Geometer's Sketchpad program and begin with a new sketch.

### **TRANSLATIONS**

1. Using the point tool, put a point on your screen.
2. Select the "Transform" menu and highlight "Translate".
3. On the "Translate" menu, bullet the rectangular vector option.
4. Input "2" in horizontal inches, hit tab, and then input "3" in vertical inches, hit the OK button.  
NOTE: Point has moved 2 inches to the right and 3 inches up.
5. Delete both points.
6. Using the line segment tool, put a line segment on your screen.
7. Using the arrow tool, highlight both endpoints and the line segment.
8. Select the "Transform" menu and highlight "Translate".
9. Input "-1" in horizontal inches, hit tab, and input "2" in vertical inches, hit the OK button.  
NOTE: Line segment has moved 1 inch to the left and 2 inches up.
10. Delete both line segments.
11. Using the line segment tool, make a triangle on your screen by drawing and connecting 3 line segments.
12. Using the arrow tool, highlight all vertices and sides of the triangle.
13. Select the "Transform" menu, highlight the "Translate" option.
14. Input "2" in horizontal inches, hit tab, input "-2" in vertical inches, hit the OK button.  
NOTE: Triangle has moved 2 inches to the right and 2 inches down.
15. Delete both triangles.

Student may practice translating any objects and use any values for their horizontal and vertical inches by using above process.

### **REFLECTIONS**

Start with a blank screen or a new sketch.

1. Using the line tool, place a horizontal line somewhere in the center of your screen. Double click on the line with the arrow tool to make it your line of reflection.
2. Using the point tool, put a point anywhere on either side of the line, and highlight it.
3. Select "Transform" menu, and highlight "Reflect".  
NOTE: Point is reflected (mirror image) to the other side of your line.
4. Delete both points.
5. Using the line segment tool, put a line segment on either side of your line and highlight the endpoints and the segment.
6. Select "Transform" menu, and highlight "Reflect".  
NOTE: Line segment is reflected to the other side of your line.
7. Delete both line segments

8. Using the line segment tool, put a triangle on either side of your line.
9. Select “Transform” menu, and highlight “Reflect”.

NOTE: Triangle is reflected to the other side of your line.

10. Delete both triangles.
11. Delete the horizontal line.

Students may place a vertical line or slanted line anywhere on your screen, double click on it to make it your line of reflection, and reflect any objects you want.

## ROTATIONS

Start with a blank screen or a new sketch.

1. Using the line segment tool, put a horizontal segment on your screen.
  2. Using the arrow tool, highlight one endpoint and the line segment.
  3. Select the “Construct” menu, and highlight “Perpendicular line”. (This draws a perpendicular line from endpoint you selected).
  4. Using the arrow tool, highlight the other endpoint and line segment again.
  5. Select the “Construct” menu, and highlight “Perpendicular line”. (This draws a perpendicular line from the other endpoint).
  6. Using the point tool, put a point anywhere on either parallel line. (Put the point in a position so you are not making a square, but a rectangle).
  7. Using the arrow tool, highlight the point you just placed and also the line it is on.
  8. Select the “Construct” menu, and highlight “Perpendicular line”.
  9. Using the arrow tool, highlight the perpendicular line and the other parallel line.
  10. Select the “Construct” menu, and highlight “Point of intersection”.
- NOTE: This process creates the 4 endpoints of a rectangle.
11. Click anywhere on the free screen.
  12. Using the arrow tool, highlight both parallel lines and the perpendicular line.
  13. Select the “Display” menu, and highlight “Hide lines”. (This will remove not erase all 3 lines from the screen).
  14. Using the arrow tool, highlight the point in the left hand corner of your rectangle, and then highlight the 3 other endpoints in a clockwise order.
  15. Select the “Construct” menu, and highlight “Line segment”. (This will draw the sides of your rectangle).
  16. Using the arrow tool, highlight 2 endpoints at opposite corners of your rectangle.
  17. Select the “Construct” menu, and highlight “Line segment”. (This will draw a diagonal in your rectangle).
  18. Repeat the last two steps to draw the other diagonal.
  19. Using the arrow tool, highlight both diagonals.
  20. Select the “Construct” menu, and highlight “Point of intersection”. (This determines the center of the rectangle).

21. Using the arrow tool, double click on the center point (This designates the point of rotation).
22. Using the arrow tool, highlight all 4 endpoints and all 4 sides of the rectangle and the center point. (Do not highlight the diagonals).
23. Select the "Transform" menu, highlight "Rotate".
24. On the Rotate screen, bullet "By Fixed Angle" option and input "45" (degrees) into the "By" window, and hit OK.  
NOTE: This will rotate the rectangle 45 degrees about the center point.
25. Delete the rotated triangle.

Student can repeat the rotation using different angles and can also change the point of rotation.

Clear screen by deleting all figures and shut down computer properly. Thank you for your cooperation.

## STUDENT HOMEWORK ASSIGNMENT

Directions: Complete the following twelve problems on the graph paper provided.

### PART I: REFLECTIONS

Problem # 1--Plot the following three vertices of a triangle on Graph # 1 and reflect the triangle about the Y-axis. Vertices: A (-1, -3), B (-5, -1), and C ( -2, 5).

Problem # 2--Plot the following three vertices of a triangle on Graph # 2 and reflect the triangle about the X-axis. Vertices: A (-4, 2), B (-3, 7) , and C (5, 6 ).

Problem # 3--Plot the following four vertices of a quadrilateral on Graph # 3 and reflect the object about the Y-axis and X-axis. Vertices: A (-4, 0 ), B (-6, 0 ), C ( 0, 5 ), and D ( 0, 2 ).

### PART II: TRANSLATIONS

Problem # 4--Sketch the given line segment under the translation  $\langle 5, -2 \rangle$

Problem # 5--Sketch the given triangle under the translation  $\langle -3, -4 \rangle$  .

Problem # 6--Sketch the given quadrilateral under the translation  $\langle 1, 6 \rangle$

### PART III: ROTATIONS

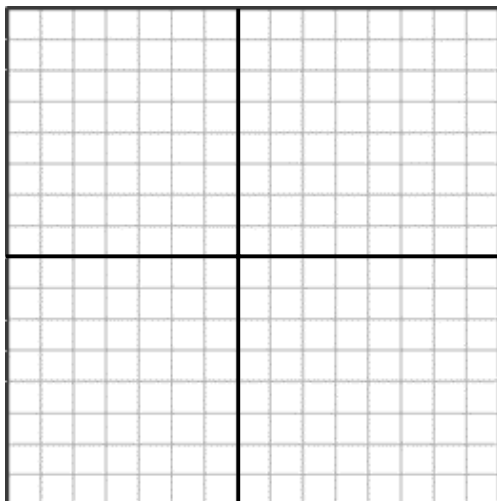
Problem # 7--Rotate the figure on graph # 7 the given number of degrees about point P and label the vertices.

Problem # 8--Rotate the figure on graph # 8 the given number of degrees about point P and label the vertices.

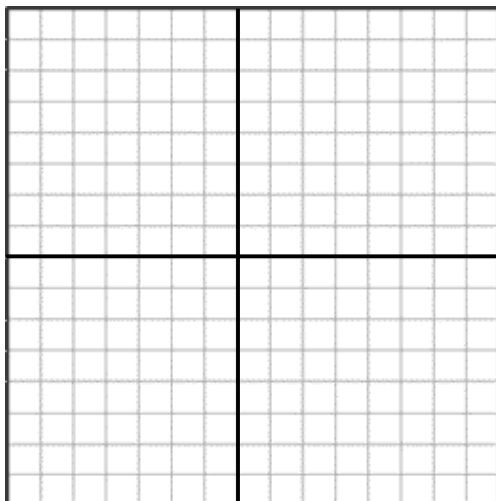
Problem # 9--Rotate the figure on graph # 9 the given number of degrees about point P and label the vertices.

GOOD LUCK !!

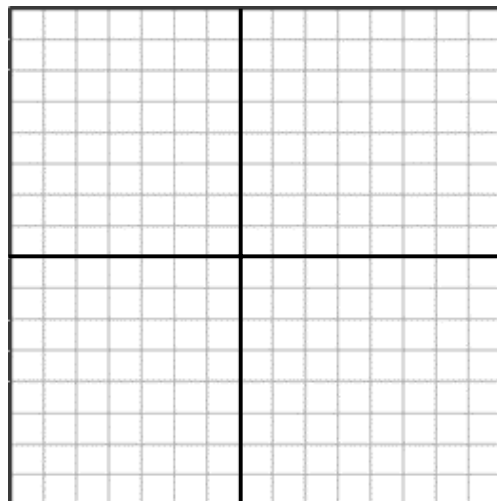
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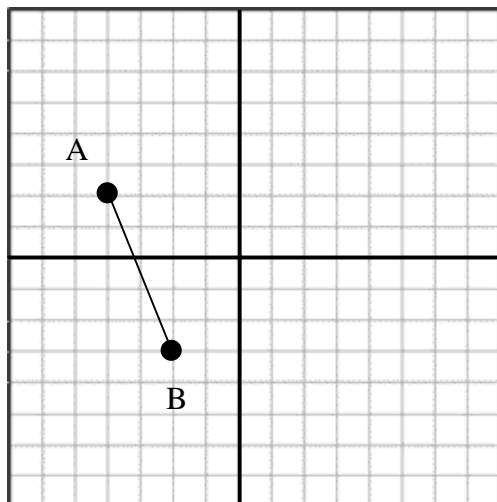
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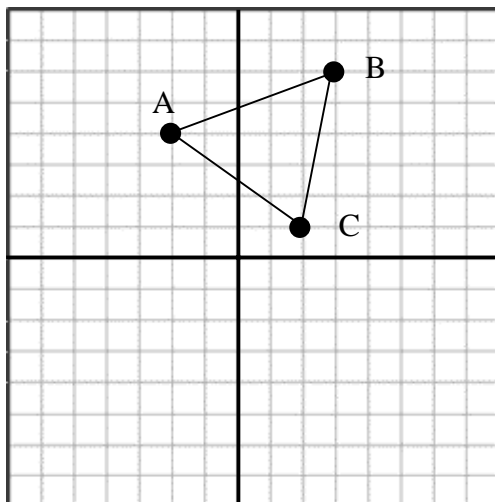
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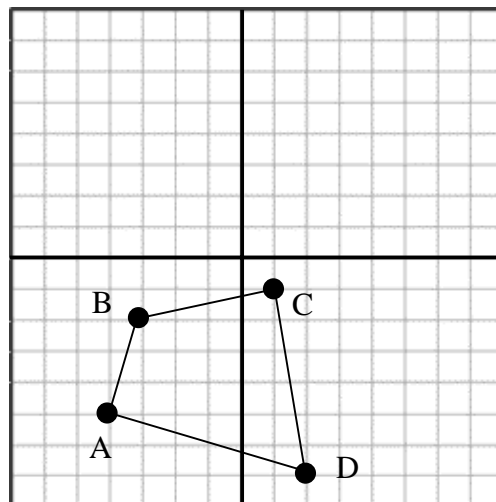
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5

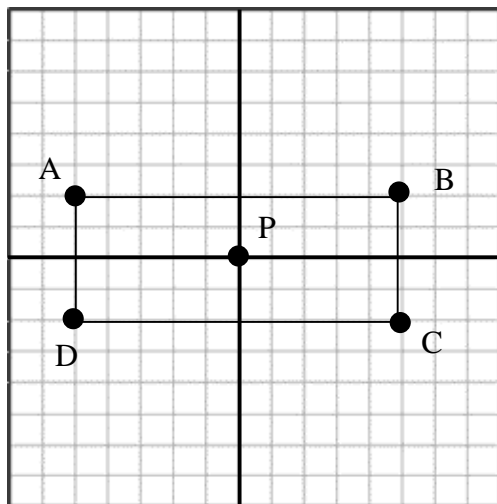


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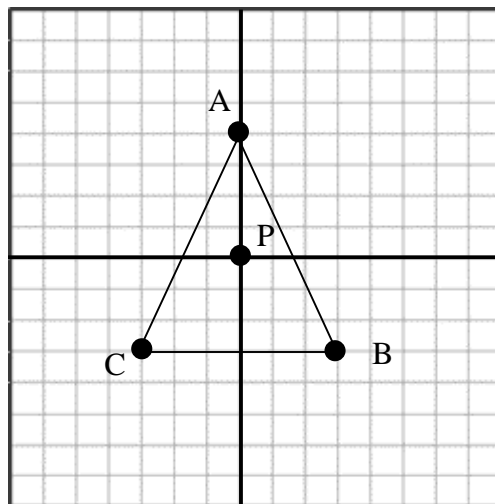
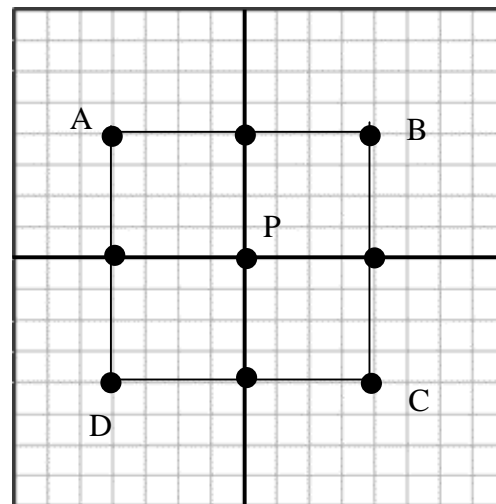
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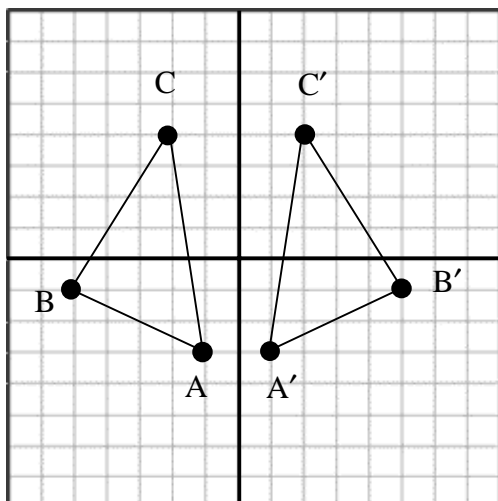


8

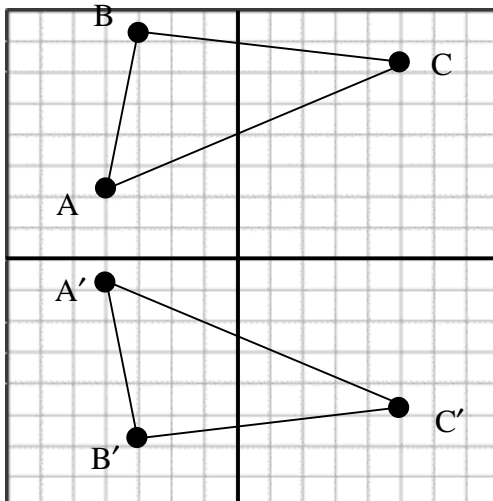
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9 Rotate counter clockwise  $45^\circ$ 

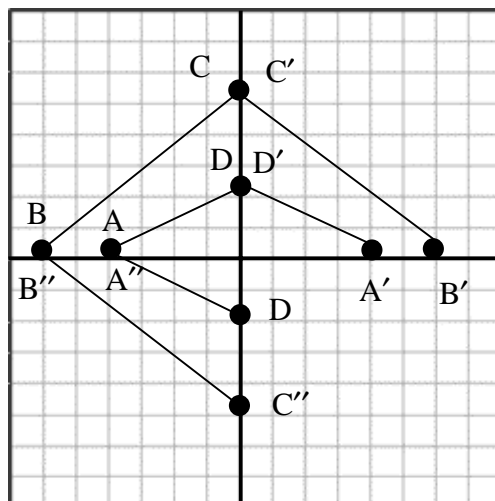
1



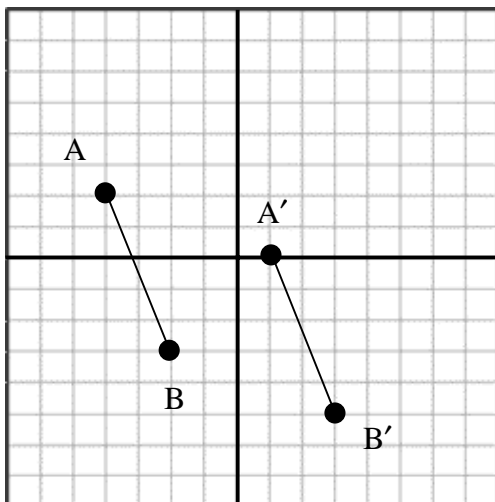
2



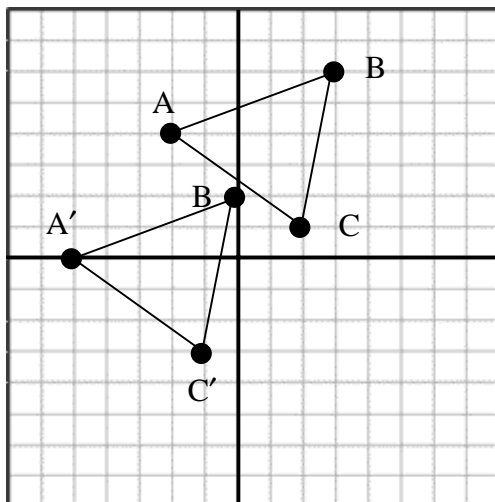
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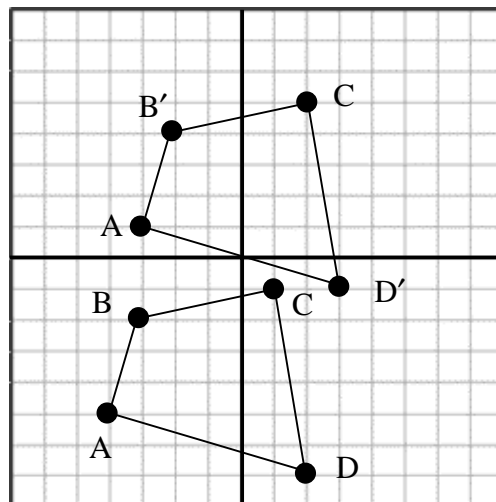
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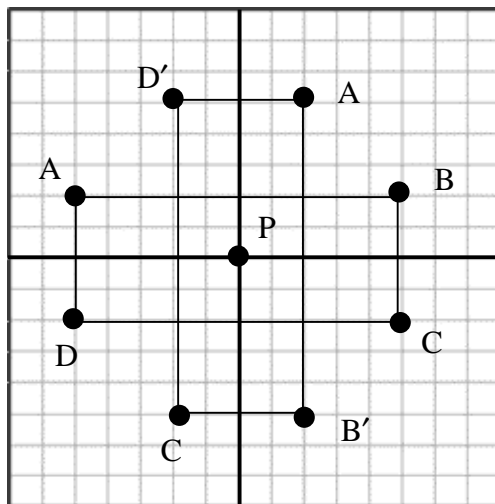
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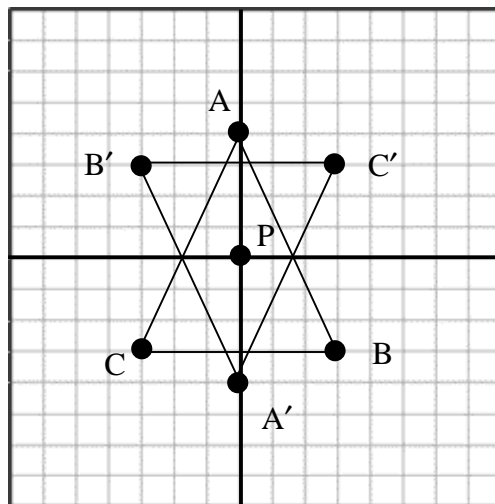
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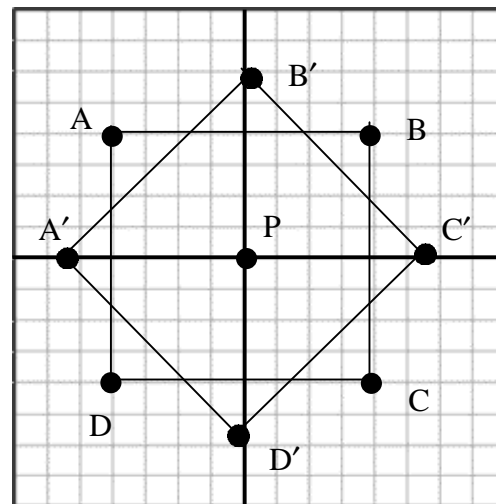
7



8



9 Rotate counter clockwise 45°





## Student Assessment

**Directions:** Read the statement and select the best response.

1. Translations are sometimes called \_\_\_\_\_.

- a. slides
- b. turns
- c. transformers
- d. flips

2. Rotations are sometimes called \_\_\_\_\_.

- a. slides
- b. turns
- c. transformers
- d. concerns

3. Reflections are sometimes called \_\_\_\_\_.

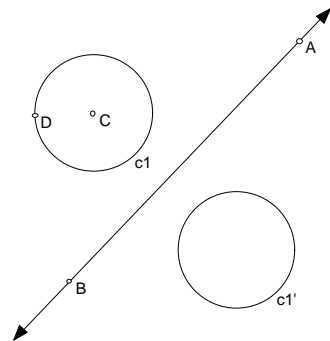
- a. slides
- b. turns
- c. transformers
- d. flips

4. Transformations are to translations as:

- a. cars are to busses
- b. books are to libraries
- c. pens are to pencils
- d. cats are to tigers

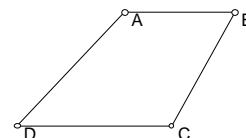
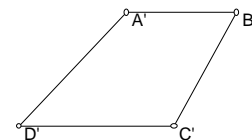
5. The figure at the right is an example of a

- a. rotation
- b. translation
- c. dilation
- d. reflection



6. The figure at the right is an example of a

- a. rotation
- b. translation
- c. dilation
- d. reflection

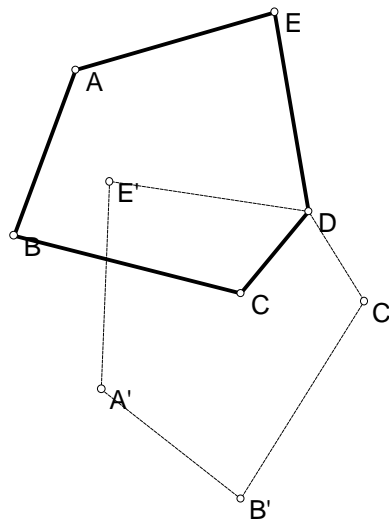


### Student Assessment (p.2)

**Directions:** Read the statement and select the best response.

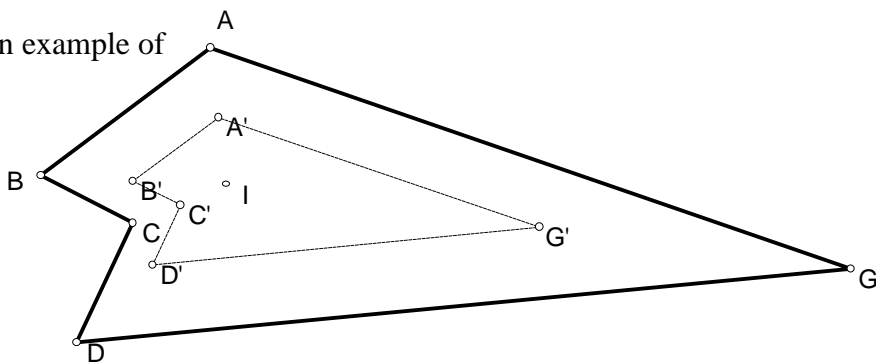
7. The figure at the right is an example of a

- a. rotation
- b. translation
- c. dilation
- d. reflection

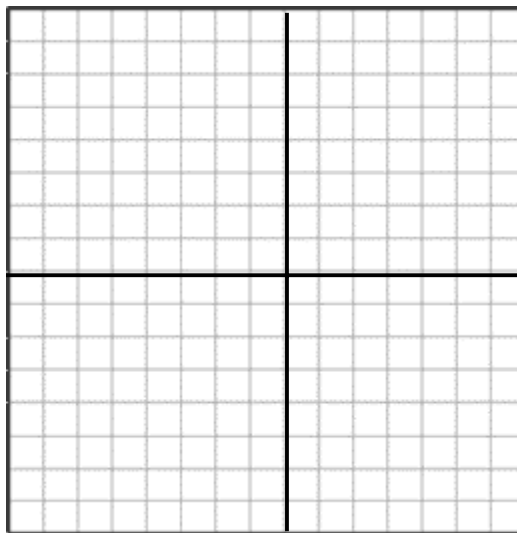


8. The figure at the right is an example of

- a. rotation
- b. translation
- c. dilation
- d. reflection

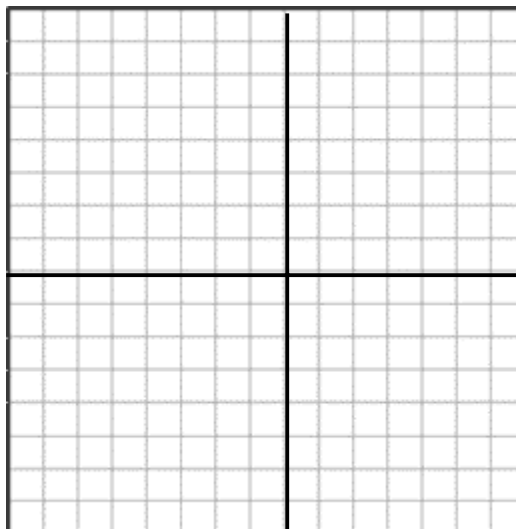


9. Graph the quadrilateral with the vertices  $A(-6,4)$ ;  $B(-3,4)$ ;  $C(-1,1)$ ; and  $D(-6,1)$  and rotate the quadrilateral  $-90$  degrees about point D.

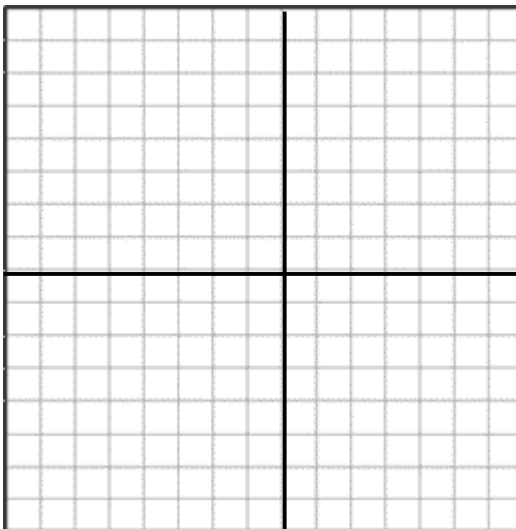


### Student Assessment (p.3)

10. Graph the triangle with vertices  $A(-4,-1)$ ;  $B(-4,-6)$  and  $C(-2,3)$  and reflect the triangle about the X-axis and label the image Triangle  $A'B'C'$ .



11. Graph the circle R with center at  $(0,0)$  and radius 6 then dilate the circle to a scale factor of 1 to 3.



## Student Assessment (Answers)

**Directions:** Read the statement and select the best response.

1. Translations are sometimes called \_\_\_\_\_.

- a. *slides*
- b. turns
- c. transformers
- d. flips

2. Rotations are sometimes called \_\_\_\_\_.

- a. slides
- b. *turns*
- c. transformers
- d. concerns

3. Reflections are sometimes called \_\_\_\_\_.

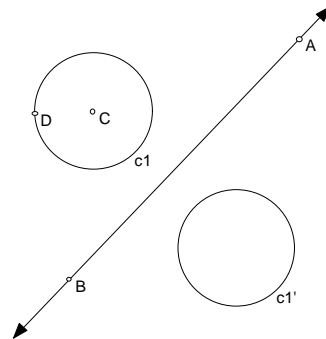
- a. slides
- b. turns
- c. transformers
- d. *flips*

4. Transformations are to translations as:

- a. cars are to busses
- b. books are to libraries
- c. pens are to pencils
- d. *cats are to tigers*

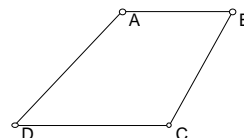
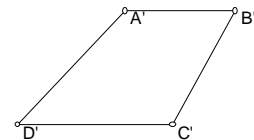
5. The figure at the right is an example of a

- a. rotation
- b. translation
- c. dilation
- d. *reflection*



6. The figure at the right is an example of a

- a. rotation
- b. *translation*
- c. dilation
- d. reflection

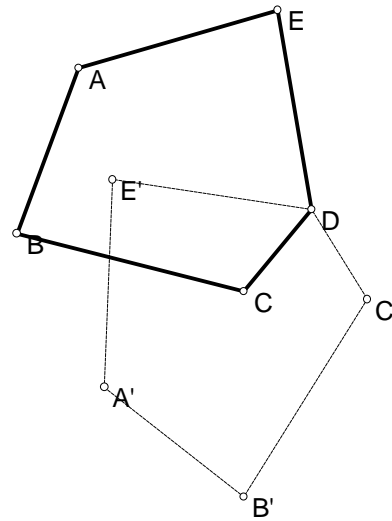


## Student Assessment (p.2 Answers)

**Directions:** Read the statement and select the best response.

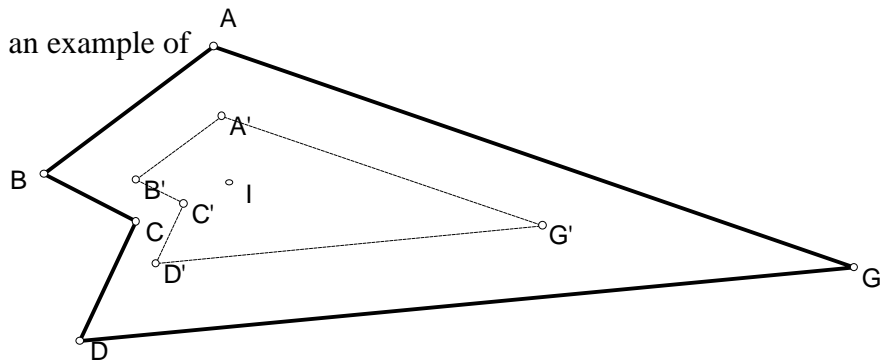
7. The figure at the right is an example of a

- a. *rotation*
- b. translation
- c. dilation
- d. reflection

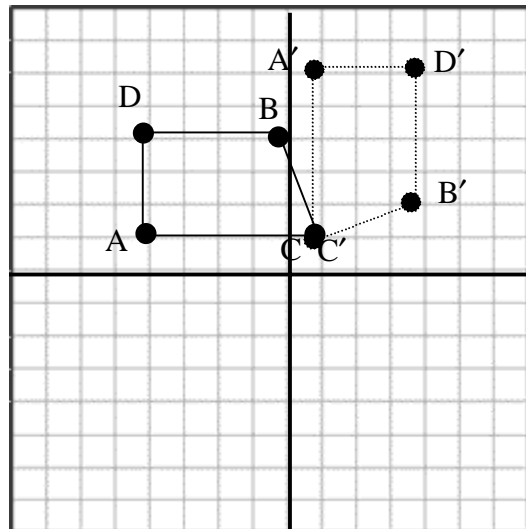


8. The figure at the right is an example of

- a. rotation
- b. translation
- c. *dilation*
- d. reflection

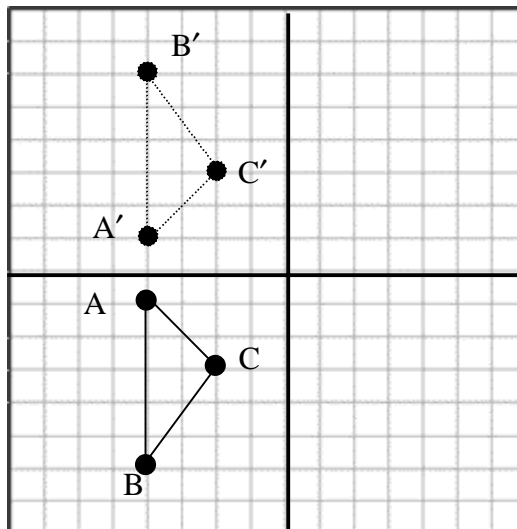


9. Graph the quadrilateral with the vertices  $A(-6,4)$ ;  $B(-2,4)$ ;  $C(-1,1)$ ; and  $D(-6,1)$  and rotate the quadrilateral  $-90$  degrees about point D.

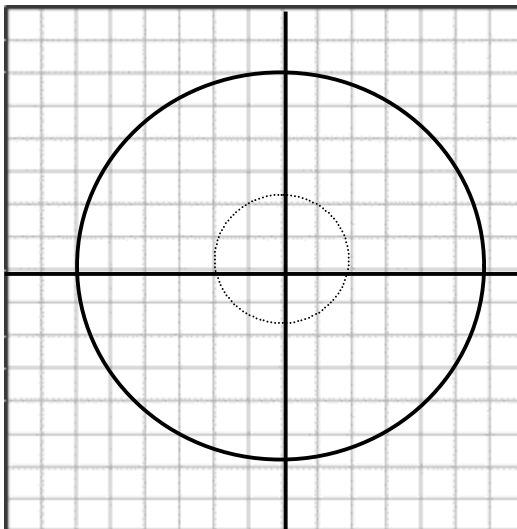


**Student Assessment (p. 3 Answers))**

10. Graph the triangle with vertices  $A(-4,-1)$ ;  $B(-4,-6)$  and  $C(-2,-3)$  and reflect the triangle about the X-axis and label the image Triangle  $A'B'C'$ .



11. Graph the circle R with center at (0,0) and radius 6 then dilate the circle to a scale factor of 1 to 3.



***Rubric for #9-11***

- 3- Graphs are complete and accurate (labels etc.).
- 2- Graphs are complete and mostly accurate.
- 1- Graphs are mostly complete and mostly accurate.
- 0- Graphs are incomplete and inaccurate.

